On the statistics of resonances and non-orthogonal eigenfunctions in a model for single-channel chaotic scattering

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We describe analytical and numerical results on the statistical properties of complex eigenvalues and the corresponding non-orthogonal eigenvectors for non-Hermitian random matrices modeling one-channel quantum-chaotic scattering in systems with broken time-reversal invariance.

The statistical properties of non-orthogonal eigenvectors of large non-selfadjoint random matrices have recently been characterised in Refs. 1,2,3,4,5.

Correlations of non-orthogonal eigenvectors are expected to determine dynamical properties of classical random systems described by non-selfadjoint operators, such as Fokker-Planck operators[6] for example; they also play an important role in quantum systems: in Ref. 3 it was observed that the statistics of non-orthogonal eigenvectors determines the properties of random lasing media. This has led to an increased interest in eigenvector statistics in non-selfadjoint random matrix ensembles (see also Ref. 7).

In a model for quantum-chaotic scattering, the complex eigenvalues $\mathcal{E}_k\,, k = 1, ..., N$ of a random $N\times N$ non-Hermitian matrix (the so-called "effective Hamiltonian") $\mathcal{H}_N = \hat{H} - i\hat{\Gamma}$ are used to describe generic statistical properties of resonances in quantum chaotic scattering (see Ref. 8 and references therein): for systems with broken time-reversal invariance (anti-unitary symmetry), the matrices $\hat{H}$ are random $N\times N$ matrices from the Gaussian Unitary Ensemble[9] with joint probability density $P(H)\,dH \propto \exp[-(N/2)\mathrm{Tr}H^2]\,dH$. In the limit of large N, the mean eigenvalue density $\nu(E)$ for such matrices is given by the semicircular law $\nu(E) = (2\pi)^{-1}\sqrt{4-E^2}$ for $|E| < 2$ (and zero otherwise). The corresponding mean spacing between neigbouring eigenvalues around the point E in the spectrum is given by $\Delta(E) = 1/[N\nu(E)]$.

The Hermitian matrices $\hat{H}$ describe the energy-level statistics of the closed counterpart of the scattering system; the Hermitian $N\times N$ matrix $\hat{\Gamma} > 0$ models the coupling of the system to scattering continua via $M = 1, 2, ...$ open channels. It has rank $M \leq N$. For our purposes it can be chosen diagonal: $\hat{\Gamma} = \mathrm{diag}(\gamma_1, \gamma_2, ..., \gamma_M, 0, ..., 0)$. The constants $0 < \gamma_c < \infty$ parametrise the strength of the coupling to the scattering continua via a given channel $c = 1, ..., M$. Here $\gamma_c = 0$ corresponds to a closed channel c, and $\gamma_c = 1$ describes the so-called perfectly coupled channel. Empirical situations correspond to the regime of large N, with M fixed and $M \ll N$. Then the widths $\Gamma_k = 2\,\mathrm{Im}\,\mathcal{E}_k$ are of the same order $1/N$ as the mean spacing $\Delta(E)$ between the positions of the neighbouring resonances along the real energy axis. In this regime, the resonances may partly or considerably overlap and first-order perturbation theory valid for small resonance overlaps breaks down. Similarly, self-consistent perturbation schemes[1,2,5,6] assuming many channels and strongly overlapping resonances are inapplicable.

A detailed analytical understanding of the statistical properties of the resonances in the regime of partial overlap has recently been achieved for the case of systems with broken time reversal invariance[8,10]. These results, based on the random matrix approach, are expected to be applicable to a broad class of quantum-chaotic systems. Indeed, the distribution of the widths Γ_k derived in Ref. 8 is in good agreement with available numerical data for quite diverse models of quantum chaotic scattering[11,12].

Much less is known on properties of non-orthogonal eigenvectors. Let $|R_k\rangle$ and $\langle L_k|$ denote the right and the left eigenvectors of the matrix $\hat{\mathcal{H}}$ corresponding to the eigenvalue $\mathcal{E}_k \equiv E_k - iY_k = E_k - i\Gamma_k/2$,

$$\begin{aligned} \mathcal{H}|R_k\rangle &= \mathcal{E}_k|R_k\rangle\,, \quad \langle L_k|\mathcal{H} = \langle L_k|\mathcal{E}_k \\ \mathcal{H}^\dagger|L_k\rangle &= \mathcal{E}_k^*|L_k\rangle\,, \quad \langle R_k|\mathcal{H}^\dagger = \langle R_k|\mathcal{E}_k^* \end{aligned} \tag{1}$$

where the symbols $\dagger$ and * stand for Hermitian conjugation and complex conjugation, respectively. Except for a set of measure zero, the eigenvalues are non-degenerate. In this case the eigenvectors form a complete, bi-orthogonal set. They can be normalised to satisfy $\langle L_k|R_l\rangle = \delta_{kl}\,$. The most natural way to characterise the non-orthogonality of eigenvectors is to consider statistics of the overlap matrix $\mathcal{O}_{kl} = \langle L_k|L_l\rangle\langle R_l|R_k\rangle$. This matrix features in two-point correlation functions in non-Hermitian systems, e.g. in description of the particle escape from the scattering region ("norm leakage", see Ref. 13).

Following Ref. 1, consider two correlation functions: a diagonal one

$$O(\mathcal{E}) = \Big\langle \frac{1}{N}\sum_n \mathcal{O}_{nn}\,\delta\left(\mathcal{E}-\mathcal{E}_k\right)\Big\rangle_{\mathcal{H}_N} \tag{2}$$

and an off-diagonal one

$$O(\mathcal{E}_1,\mathcal{E}_2) = \Big\langle \frac{1}{N}\sum_{n\neq m}\mathcal{O}_{nm}\;\delta(\mathcal{E}_1-\mathcal{E}_n)\;\delta(\mathcal{E}_2-\mathcal{E}_m)\Big\rangle_{\mathcal{H}_N}. \tag{3}$$

Here $\langle\cdots\rangle_{\mathcal{H}_N}$ stands for an ensemble average over $\mathcal{H}_N$. The correlation functions (2,3) characterise the average

non-orthogonality of eigenvectors corresponding to resonances whose positions in the complex plane are close to the complex energies $\mathcal{E}$, and $\mathcal{E}_1, \mathcal{E}_2$. Here $\delta(\mathcal{E})$ stands for a two-dimensional $\delta-$function of the complex variable $\mathcal{E}$.

In the context of lasing media, the diagonal correlator (2) characterises average excess noise factors (Petermann factors), and the off-diagonal correlator (3) describes average cross correlations between thermal or quantum noise emitted into different eigenmodes[14]. Note that for any ensemble with orthogonal eigenvectors and complex eigenvalues $\mathcal{E}$ (for normal matrices), $O(\mathcal{E})$ is equal to the mean density of complex eigenvalues, and the off-diagonal correlator vanishes: $O(\mathcal{E}_1, \mathcal{E}_2) \equiv 0$.

Both diagonal and off-diagonal eigenvector correlators were introduced and calculated for the case of Ginibre's ensemble of non-Hermitian matrices in Ref. 1. For the ensemble $\mathcal{H}_N$ pertinent to chaotic scattering, both types of eigenvector correlators were found recently for the regime of very strongly overlapping resonances when widths typically much exceed the mean separation[2,5]. Physically this regime corresponds to a situation where the scattering system is coupled to the continuum via a large number $M \gg 1$ of open channels[8]. In this case the self-consistent Born approximation is adequate[1,2,5,6], a perturbative approximation valid in the limit of large N, large M, and $|\mathcal{E}_1 - \mathcal{E}_2| \neq 0$, provided $\mathcal{E}_1, \mathcal{E}_2$ are well inside the support of the spectrum. A non-perturbative expression for the diagonal correlator $O(z)$ valid for any number of open channels was obtained in Ref. 3 by employing a heuristic analytic continuation procedure. For the case of the resonance widths, this heuristic scheme is known to reproduce the exact expression[8]. It is thus natural to expect that this procedure is adequate in the case of eigenvector statistics, too, although this remains to be proven.

No non-perturbative results for the off-diagonal eigenvalue correlator $O(\mathcal{E}_1, \mathcal{E}_2)$ have so far been reported, to the best of our knowledge.

In the present paper we provide exact non-perturbative expressions for both diagonal and off-diagonal eigenvector correlators valid for the case of a system with broken time-reversal invariance (anti-unitary symmetry) coupled to continuum via a single open channel ($M = 1$) with coupling strength γ. The single-channel case describes pure resonant chaotic reflection. This case is more amenable to analytical treatment than a general case ($M > 1$), combining both reflection and transmission phenomena. Understanding the single-channel case should be considered as a useful step towards a more complete picture[15].

Our result for the diagonal correlator is

$$O(\mathcal{E}) = \nu e^{-4\pi g Y/\Delta} \frac{d}{dY} \left\{ e^{2\pi g Y/\Delta} \frac{\sinh(2\pi Y/\Delta)}{2\pi Y/\Delta} \right\} \qquad (4)$$

where $\mathcal{E} = E - iY$, $\nu \equiv \nu(E)$, $\Delta = \Delta(E)$ and $g = (\gamma + \gamma^{-1})/(2\pi\nu)$ is the effective (renormalised) coupling strength. The result for $O(\mathcal{E})$ agrees with one

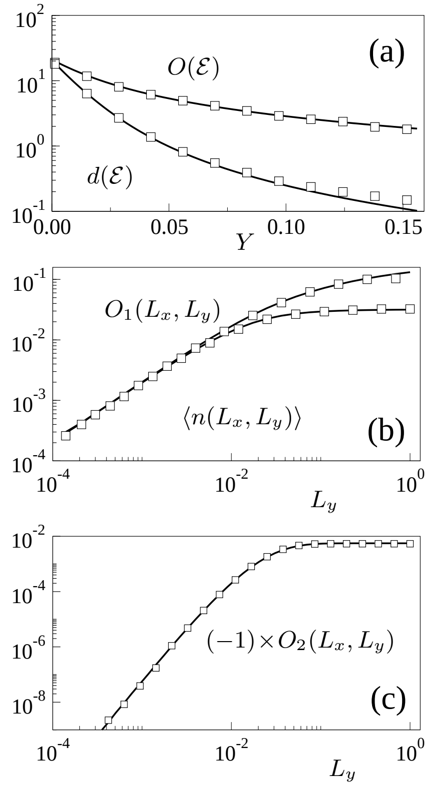

FIG. 1: Numerical (□) and analytical results (solid line) for (a) $d(\mathcal{E})$ and $O(\mathcal{E})$ as a function of Y for $N = 32$, and $\gamma = 0.9$; (b) $\langle n(L_x, L_y)\rangle$ and $O_1(L_x, L_y)$ for $N = 32$, $L_x = 0.1$, and $\gamma = 0.9$; (c) $O_2(L_x, L_y)$ for $N = 32$, $L_x = 0.1$, and $\gamma = 0.9$.

reported in Ref. 3 confirming the validity of the analytical continuation scheme used there. For the sake of comparison we present here also the expression for the single-channel resonance density defined as $d(\mathcal{E}) = \left\langle N^{-1} \sum_k \delta\left(\mathcal{E} - \mathcal{E}_k\right)\right\rangle_{\mathcal{H}_N}$ and given by[8]:

$$d(\mathcal{E}) = -\nu \frac{d}{dY} \left\{ e^{-2\pi g Y/\Delta} \frac{\sinh(2\pi Y/\Delta)}{2\pi Y/\Delta} \right\}. \qquad (5)$$

We have compared these analytical expressions, valid in the limit $N \to \infty$, with direct numerical diagonalisations of finite-dimensional matrices $\mathcal{H}_N$, see Fig. 1(a). This is of interest since empirically, the ensemble average $\langle \cdots \rangle_{\mathcal{H}_N}$ is usually replaced by an energy average over several spectral windows, each of which may typically contain of the order of 10 or 100 resonances, corresponding to a finite value of N. We observe that the analytical results describe the numerical data well, except for small deviations at large values of Y. Numerically it is easier to compute smoothed averages, such as the mean number of eigenvalues $\langle n(L_x, L_y)\rangle$ inside a rectangular domain

$$A = \begin{cases} -L_x/2 \leq \operatorname{Re}\mathcal{E} \leq L_x/2 \\ 0 \leq \operatorname{Im}\mathcal{E} \leq L_y \end{cases} \qquad (6)$$

in the complex plane. This quantity can be obtained from the mean density $d(\mathcal{E})$ by integration over A. Similarly

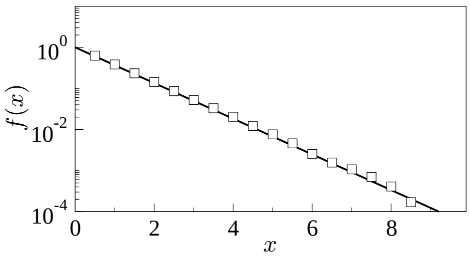

FIG. 2: Numerical (□) and analytical results (solid line) for the distribution of the narrowest resonance for $\beta = 2$, and $\gamma = 0.1$, $W = 0.2$, and $N = 128$. Here $x = \pi g n\,\Gamma/\Delta$.

one can define the function $O_1(L_x, L_y)$ as the integral of the diagonal correlator $O(\mathcal{E})$ over the same domain, obtaining $O_1(L_x, L_y) = \langle N^{-1}\sum_{\mathcal{E}_k\in A}\mathcal{O}_{kk}\rangle$. Numerical versus analytical results for these two quantities are plotted in Fig. 1(b).

For the off-diagonal correlator $O(\mathcal{E}_1, \mathcal{E}_2)$ we obtain

$$O(\mathcal{E}_1,\mathcal{E}_2) = N(\pi\nu/\Delta)^2 e^{-2\pi g(Y_1+Y_2)/\Delta} \quad (7)$$
$$\times \int_{-1}^{1} d\lambda_1 \int_{-1}^{1} d\lambda_2 (g+\lambda_1)(g+\lambda_2) e^{i\pi\Omega(\lambda_1+\lambda_2)/\Delta}$$
$$\times\ e^{-\pi Y_2(\lambda_1-\lambda_2)/\Delta}\left[e^{\pi Y_1(\lambda_1-\lambda_2)/\Delta} - e^{-\pi Y_1(\lambda_1-\lambda_2)/\Delta}\right]$$

where $\mathrm{Re}\,\mathcal{E}_{1,2} = E_{1,2} = E \mp \Omega$ and it is assumed that $\Omega \sim \Delta$. We have also calculated the corresponding smoothed average $O_2(L_x, L_y) = \langle N^{-1}\sum_{\mathcal{E}_m\neq\mathcal{E}_n\in A}\mathcal{O}_{mn}\rangle$, by integrating $\mathcal{E}_1$ and $\mathcal{E}_2$ in (7) over the domain A. In Fig. 1(c) we compare this result (valid in the limit of $N\to\infty$) with those of numerical diagonalisations of finite matrices; the agreement is good already for $N = 32$.

We have also found a way to calculate exactly the distribution $f(\Gamma)$ of the width of the most narrow resonance among those falling in a window $[E - W/2, E + W/2]$ in the vicinity of a given point E in the spectrum. Assuming that the mean number $n = W/\Delta$ of resonances is large ($n \gg 1$), but still $W \ll 1$ to preserve spectral locality (the density of states should not change significantly across the spectral window):

$$f(\Gamma) = \frac{\pi g n}{\Delta}\, e^{-\pi g n\Gamma/\Delta}\,. \quad (8)$$

This distribution is of great interest in the theory of random lasing[3]. The functional form of the distribution was found in Ref. 3 by employing plausible qualitative arguments yielding Eq. (8), but with renormalised effective coupling g replaced by its "weak coupling" limit $\gamma/2\pi\nu$. We see that the difference with exact formula amounts to the factor 2 in the exponent for the case of perfect coupling $\gamma = 1$. In Fig. 2, the result (8) is compared to results of numerical diagonalisations for $N = 128$ and $W = 0.2\Delta$, corresponding to $n \approx 8.15$.

In the remainder of this article, we outline the derivation of the results (4),(7),(8). The main idea is to use that fact that the complex eigenvalues (resonances) $\mathcal{E}_k$ are poles of the $M\times M$ scattering matrix $\hat{S}(\mathcal{E})$ in the complex energy plane. Using the standard expression for the scattering matrix in terms of the non-Hermitian Hamiltonian $\mathcal{H}_N$ (see e.g. Ref. 8) the residues corresponding to these poles can be found and after some algebraic manipulations we arrive at the following relation:

$$\mathrm{Tr}\left\{\mathrm{Res}\left[\hat{S}(\mathcal{E})\right]_{\mathcal{E}=\mathcal{E}_n}\mathrm{Res}\left[\hat{S}^\dagger(\tilde{\mathcal{E}}^*)\right]_{\tilde{\mathcal{E}}^*=\mathcal{E}^*_m}\right\} \quad (9)$$
$$= (\mathcal{E}^*_m - \mathcal{E}_n)(\mathcal{E}_n - \mathcal{E}^*_m)\ \mathcal{O}_{mn}\,.$$

This relation is valid for arbitrary M, but for $M > 1$ it appears to be of no obvious utility, due to difficulties in evaluating the ensemble average of the trace of the residues on the left-hand side. However for the case of one single open channel the scattering matrix can be written as

$$S(\mathcal{E}) = \prod_{k=1}^{N}\frac{\mathcal{E}-\mathcal{E}^*_k}{\mathcal{E}-\mathcal{E}_k}\quad,\quad S^\dagger(\mathcal{E}) = \prod_{k=1}^{N}\frac{\mathcal{E}^*-\mathcal{E}_k}{\mathcal{E}^*-\mathcal{E}^*_k} \quad (10)$$

which follows, up to an irrelevant "non-resonant" phase factor, from the requirement of $S-$ matrix analyticity in the upper half-plane and unitarity for real energies. Substituting Eq. (10) into Eq. (9) yields the relation:

$$\mathcal{O}_{mn} = \frac{(\mathcal{E}_n-\mathcal{E}^*_n)(\mathcal{E}_m-\mathcal{E}^*_m)}{(\mathcal{E}_n-\mathcal{E}^*_m)^2}\prod_{k\neq n}^{N}\frac{\mathcal{E}_n-\mathcal{E}^*_k}{\mathcal{E}_n-\mathcal{E}_k}\prod_{k\neq m}^{N}\frac{\mathcal{E}^*_m-\mathcal{E}_k}{\mathcal{E}^*_m-\mathcal{E}^*_k} \quad (11)$$

expressing the eigenvector overlap matrix in terms of complex eigenvalues $\mathcal{E}_k$[16]. This gives a possibility to find the diagonal and off-diagonal correlators, Eqs. (2,3), by averaging $\mathcal{O}_{mn}$ over known joint probability density of complex eigenvalues[10] for the single-channel scattering system:

$$\mathcal{P}(\mathcal{E}_1, ..., \mathcal{E}_N) = \frac{e^{\frac{-N}{2}\gamma^2}}{\gamma^{N-1}}|\Delta\{\mathcal{E}_1, ..., \mathcal{E}_N\}|^2 \quad (12)$$
$$\times\ e^{-\frac{N}{4}\sum_k(\mathcal{E}_k^2+\mathcal{E}_k^{*2})}\delta\Big(\gamma - \sum_k \mathrm{Im}\mathcal{E}_k\Big)\,.$$

Using this expression one may notice that

$$O(\mathcal{E}) = \frac{\tilde{\gamma_1}^{N-2}}{\gamma^{N-1}}e^{-\frac{1}{2}\left[N\gamma^2-(N-1)\tilde{\gamma_1}\right]}e^{-\frac{N}{4}(\mathcal{E}^2+\mathcal{E}^{*2})} \quad (13)$$
$$\times\left\langle\det\left(\mathcal{E}-\mathcal{H}^\dagger\right)\left(\mathcal{E}^*-\mathcal{H}\right)\right\rangle_{\tilde{\mathcal{H}}_{N-1}}$$

where $\tilde{\mathcal{H}}_{N-1}$ stands for the non-Hermitian matrix $\mathcal{H}$ of the same type as $\mathcal{H}_N$ but of the lesser size $(N-1)\times(N-1)$, and with coupling γ replaced by a modified coupling $\tilde{\gamma_1} = \gamma - \mathrm{Im}\,\mathcal{E}$. Analogously

$$O(\mathcal{E}_1,\mathcal{E}_2) = \frac{\tilde{\gamma_2}^{N-3}}{\gamma^{N-1}}e^{-\frac{1}{2}\left[N\gamma^2-(N-2)\tilde{\gamma_2}\right]} \quad (14)$$
$$\times e^{-\frac{N}{4}\sum_{n=1}^{2}(\mathcal{E}_n^2+\mathcal{E}_n^{*2})}(\mathcal{E}_1-\mathcal{E}^*_1)(\mathcal{E}_2-\mathcal{E}^*_2)$$
$$\times\left\langle\det\left(\mathcal{E}_1-\mathcal{H}^\dagger\right)\left(\mathcal{E}^*_1-\mathcal{H}^\dagger\right)\left(\mathcal{E}_2-\mathcal{H}\right)\left(\mathcal{E}^*_2-\mathcal{H}\right)\right\rangle_{\tilde{\mathcal{H}}_{N-2}}$$

where $\tilde{\mathcal{H}}_{N-2}$ is of the size $(N-2)\times(N-2)$, and with coupling γ replaced by a modified coupling $\tilde{\gamma_2} = \gamma - \mathrm{Im}\,\mathcal{E}_1 - \mathrm{Im}\,\mathcal{E}_2$.

In this way the problem is reduced to calculating a correlation function of characteristic polynomials of large non-Hermitian matrices. A closely related object was calculated in Ref. 10, and we can adopt those methods to our case. The scaling limit $N \gg 1$ such that $\mathrm{Im}\,\mathcal{E}_{1,2} = \Gamma_{1,2} \sim 2\Omega = \mathrm{Re}\,(\mathcal{E}_1 - \mathcal{E}_2) \sim \Delta \propto N^{-1}$ of the resulting expressions yields the formulas Eqs. (4)-(7) above.

Let us briefly comment on a way of calculating the distribution Eq. (8) of the widths of the most narrow resonance in a given window. Instead of extracting such a quantity from the joint probability density Eq. (12) we find it more convenient to consider

$$\mathcal{P}\,(z_1,...,z_n) \propto \frac{1}{T^{n-1}}|\Delta\{z_1,...,z_n\}|^2 \times\delta\Big(1-T-\prod_{k=1}^{n}\mathrm{Im}|z_k|^2\Big) \qquad (15)$$

defined for complex variables $z_i = r_i e^{\theta_i}$ inside the unit circle: $r_i = |z_i| \leq 1$. For $0 \leq T \leq 1$ this formula has interpretation of the joint probability density of complex eigenvalues z_i for the ensemble of $n\times n$ subunitary matrices and is a very natural "circular" analogue of Eq. (12). The similarity is in no way a superficial one, but rather has deep roots in the theory of scattering[17]. The parameter T controls the deviation of the corresponding matrices from unitarity, much in the same way as the parameter γ controls the deviation of $\mathcal{H}_N$ from Hermiticity. More precisely, T should be associated with the renormalised coupling constant g via the relation $g = 2/T - 1$. In the limit $n \gg 1$ the eigenvalues z_k are situated in a narrow vicinity of the unit circle. Their statistics is shown[17] to be indistinguishable from that of the complex eigenenergies $\mathcal{E}$, when the latter considered *locally*, i.e. on the distances comparable with the mean spacing Δ. In particular, the distances $1 - r_i$ from the unit circle should be interpreted as the widths of the resonances.

The form of Eq. (15) allows one to integrate out the phases θ_i by noticing that:

$$\int_0^{2\pi}\frac{d\theta_1}{2\pi}\cdots\int_0^{2\pi}\frac{d\theta_n}{2\pi}\prod_{k<j}|r_k e^{i\theta_k} - r_j e^{i\theta_j}|^2 = \sum_{\{\alpha\}} r_1^{2\alpha_1}\cdots r_n^{2\alpha_n} \qquad (16)$$

where the summation goes over all possible permutations $\{\alpha\} = (\alpha_1,...,\alpha_n)$ of the set $1,...,n$ (in fact in the right-hand side we deal with the object known as "permanent", see e.g. Ref. 19). In this way we arrive at a joint probability density of the radial coordinates only, and the distribution Eq. (8) follows after a number of integrations and the limiting procedure $n \gg 1$.

In conclusion, we presented a detailed analytical and numerical investigation of statistics of resonances and associated bi- orthogonal eigenfunctions in a random matrix model of single channel chaotic scattering with broken time-reversal invariance. Among challenging problems deserving future research we would like to mention extending our results to the case of more than one channel and to time-reversal invariant systems[18], as well as the problem of understanding fluctuations of the non-orthogonality overlap matrix $\mathcal{O}_{mn}$.

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